Creating a 'Safe', Supportive Mathematics Classroom Environment

Recently I had a conversation with some colleagues who teach in a university. They were very worried about something they had noticed about their undergraduate students - a fear of making mistakes. They had noticed that these students were very reluctant to hand in assignments in case they were 'wrong' and were often spending time very unproductively in checking and re-checking their answers. While it is, of course, important to encourage students to be careful about checking their work, and to help them to develop a repertoire of checking strategies, this conversation does seem to reflect a growing problem, that more and more students are becoming afraid to try new things in case they fail, and/or become depressed and question their own self-worth if they do make mistakes. Mathematics, with its emphasis on 'right' or 'wrong' answers can potentially reinforce these fears. On the other hand, however, the mathematics classroom can also be the perfect environment for sensitive teachers to help their pupils to face up to and overcome these fears - and, of course, the earlier in the child's school life that this support begins, the better.

The purpose of this article is to illustrate some ways in which mathematics teachers can help to create a secure, supportive classroom environment in which the pupils learn to not fear failure and to value making mistakes as an opportunity to learn and grow. Each section begins with a quotation from Sathya Sai Baba.

"True education should make a person compassionate and humane."

It is likely that unwillingness to participate in the mathematics classroom arises from lack of *understanding and compassion*, which can often be unconscious, by teachers and other pupils. Consequently, we need to ask the question: how can we encourage more effective participation by any students not participating fully?

- Do not be angry if a child cannot understand something or makes a mistake, because this can lead to fear of failure.
- Show them how to recover from the mistake and try again.
- Tell them about famous people who were not afraid to make mistakes (see stories below), or about some of the mistakes you have made - but also encourage accuracy and patiently ask them to correct their careless errors. A useful source of ideas is a book called "Mistakes That Worked" by Charlotte Foltz Jones.

"Students should not allow success or failure to ruffle their minds unduly. Courage and self-confidence must be instilled in the students."

- Use positive visual and body-language cues (nodding, smiling) and prompts (ah ha, hmm) to encourage them to arrive at appropriate answers.
- Be careful not to frown if a child makes a mistake, and don't allow other children to frown if a classmate makes a mistake either.

"There is over emphasis on quick and easy gains rather than patience, fortitude and hard work."

> Peter was a very clever eleven-year-old. In the final year of his primary schooling, there was only one test on which he scored less than 100%, and then he only lost half a mark. His classwork was always done quickly and correctly. When he knew that he could succeed, he was confident and willing to work hard. To challenge his thinking, Peter's teacher would give him some difficult problems. If Peter could not immediately see a way to solve a problem, he became a different child. He would sit, drawing on his notepad, or wander around the room. He would even ask his teacher if he could spend the time tidying the storeroom. Peter, who was normally so successful and confident, was afraid to tackle a difficult task because he was afraid that he might fail. So his solution was to quit, to make the fears go away. Fortunately, the story had a happy ending, because Peter and his teacher worked together to help him to develop more courage to tackle difficult problems rather than taking the easiest path of stopping.

Many writers have written about students such as Peter, who expect solutions to come to them quickly and easily and will give up rather than face negative emotions associated with trying the task. Another concern is that they often are not aware of when it is worthwhile to keep on exploring an idea and when it is appropriate to abandon it because it is leading in a wrong direction. They need to know when it is appropriate to use a particular approach to the task, and how to recover from making a wrong choice.

> Clare, aged ten, was given the following problem to solve:
>
> By changing six figures into zeros you can make this sum equal 1111.
>
> $$\begin{array}{r} 111 \\ 333 \\ 555 \\ 777 \\ +\underline{999} \\ 2775 \end{array}$$
>
> Clare selected the strategy of changing numbers in all three columns simultaneously. She worked at the task with *patience* and *fortitude* for two hours. As she worked, she said to herself, "I know that this is going to work. All I need is time, to find the right combination." After she repeated the strategy 21 times, her teacher interrupted and suggested that it might be time to look for another way to solve the problem.

In Peter's case, it was enough for his teacher to tell him that frustration, is a normal part of problem solving, and to encourage him to spend more time working on the task. Clare, on the other hand, was "overpersevering", locked into persistently pursuing one approach when it may be more appropriate when stuck to use other strategies, even such as help-seeking. One of the responsibilities of a mathematics teacher is to help pupils to learn how to persevere when the problem-solving process becomes difficult. They also need to know how to make decisions about avoiding time being wasted on "overperseverance".

STRATEGIES FOR ENHANCING PERSEVERANCE
1. Equip learners with a range of strategies/techniques for solving different types of problems.
2. Encourage them to experience the full range of positive <u>and</u> negative emotions associated with problem solving.
3. Promote the desire to persevere.
4. Help them to make "managerial" decisions about whether to persevere with a possible solution path (when to keep trying, and when to stop).
5. Encourage them to find more than one way to approach the problem.

One sequence of strategies which is used frequently by successful, persevering problem solvers is the following:

1.	Try an approach.
2.	Try it 2-3 times in case using different numbers or correcting errors might work.
3.	Try something <u>different</u>. (You might decide to come back to your old way later.)

The Appendix shows how one student used the sequence to persevere successfully with a problem.

Stories About Famous Mathematicians

When you are teaching the appropriate topic, take a minute to tell your pupils an anecdote about one of the famous mathematicians who contributed to this particular field of mathematics. It is important for pupils to be aware of the 'human' side of these famous people. "Using biographies of mathematicians can successfully bring the human story into the mahematics class. What struggles have these people undergone to be able to study mathematics?..." (Voolich, 1993, p.16)

MARY SOMERVILLE
Born 1780 in Burntisland, Scotland
<u>Examples of Contribution to Mathematics</u>: algebra, differential and integral calculus

Mary was one of the world's first famous female mathematicians. She became interested in mathematics, and desperately wanted to study it, at a time when it was not considered acceptable for a woman to do so. She bought books on algebra and geometry and read them at night. Despite disapproval from the people around her, she persisted with her struggle to learn. Later in her life she began to solve problems in a magazine, and won a prize for her solution to an algebra problem. She went on to write several books about mathematics and science. Later in her life, she reflected on "the long course of years in which I had persevered almost without hope. It taught me never to despair" (p.6).

Perl (1993)

MARIA AGNESI
(1718-1799) Italy
Example of Contribution to Mathematics: calculus

"Maria was a child prodigy, but was also shy. She stayed at home, teaching the younger children and following her own studies. When her mother died after giving birth to twenty-one children, Maria took over the running of the household.

At the age of twenty she started a ten-year project, a book bringing together the work on calculus of Leibnitz and Newton titled Analytic Institutions. Sometimes she would have trouble with a problem. But her mind went on working even in her sleep; she would sleepwalk to her study and back to bed. In the morning, she would find the answer to the problem waiting on her desk. Her book made her famous; she was living proof of what she had argued at nine years old [that women had a right to study science].

But Maria had other interests in her life apart from mathematics. She had always worked with the poor people in her area, and she had asked her father for separate rooms and turned them into a private hospital. She worked at the hospital (and another) until she died at the age of eighty-one.

Maria Agnesi wrote an important book on mathematics, as well as another unpublished book. She ran a household of over twenty people, and she worked for people who had not had her luck and opportunities. Each one of these things was remarkable, but she did them all."

(Lovitt and Clarke, 1992, p.560)

"Education should impart to students the capacity or grit to face the challenges of daily life."

For students who have tried but are still having difficulties, McDonough (1984) advised that the teacher:

- ◊ ask the pupils to restate the problem in their own words and if this indicates that they have mis-read or mis-interpreted the instructions, ask them to read the instructions again,

- ◊ to help with the understanding of the written instructions question the pupils carefully to find out if they know the meanings of particular words and phrases (i.e. mathematical terminology),

- ◊ have the pupils <u>show</u> the teacher what they have done, compare this to what is asked in the instructions, and question the pupils to see if they could think of another method, for example, "Could you have done this another way?" or, "Have you ever done a task like this before?"

- ◊ if necessary, give the children a small hint <u>but</u> only after questioning them carefully to find out what stage they have reached.

- If the teacher follows procedures such as those described above, the pupils will be encouraged to be more thoughtful and self-reliant.

- If pupils are panicking or unable to think what to do, introduce them to the valuable technique of silent sitting - that is, sitting for a few minutes in a state of complete outer <u>and</u> inner silence. You can tell them about famous mathematicians who have solved problems by using this technique.

SIR ISAAC NEWTON

"We all have something within us which helps us, guides us, gives us the conscience to know what is right and wrong. This "something" also gives us knowledge and wisdom. Whenever we cannot think of a solution to a problem we sit still and calm our mind. Very often the answer will come in a moment of intuition. Sir Isaac Newton, after thinking for some time on the effect of gravity, could not solve the problem. So Newton went for a walk to relax and when sitting quietly under an apple tree, saw an apple fall down; in a flash of understanding Newton understood the law of gravity which governs the movement of minute particles as well as the stars and planets. Many great scientific discoveries have been made not during serious thinking or when doing a lot of calculations but while the mind is relaxed. This is when intuition starts."

MARY SOMERVILLE
Born 1780 in Burntisland, Scotland
<u>Examples of Contribution to Mathematics</u>: algebra, differential and integral calculus

> (continued from above)
> "Mary Somerville used an approach to her work that is useful today. If she couldn't find the key to unlock a difficult problem she stopped work and turned to the piano, her needlework, or a walk outdoors. Afterward, she returned to the problem with her mind refreshed and could find the solution. If she could not understand a passage in her reading, she would read on for several pages. Then, going back, she could often understand what was meant in the part which had been confusing" (p.12).
>
> Perl (1993)

Education must award self-confidence, the courage to depend on one's own strength.

- Some of us may believe that it is acceptable to be untruthful if it is to avoid hurting somebody else's feelings. On the other hand, some people can also be cruelly truthful and blunt if they do not like something about another person. We need to realise that neither of these behaviours is really appropriate.

- If we are patient and consistent in our approach and give criticism with compassion, we will have a more significant influence on the child's subconscious levels of thinking than we realise.

This does not mean that you have to be blunt or to hurt somebody else's feelings by telling them something unkind. For example, when correcting students you could say, "I don't like the way you answered that question. I like it better when you give me a sensible answer and I know that you have put thought into it." Or you could say, "I don't really like the way you have done this piece of work. I prefer it when you do it more slowly and make fewer mistakes". This means that you are making it very clear to the other person why you are not happy and how you would prefer her to behave.

By example and precept, in the classroom and the playground, the excellence of intelligent co-operation, of sacrifice for the team, of sympathy for the less gifted, of help…has to be emphasised.

MARY EVERETT BOOLE
Born 1832 in England and lived in Poissy, France as a child
<u>Examples of Contribution to Mathematics</u>:
geometry of angles and space; string geometry (curve stitching), mathematical psychology (understanding how people learn mathematics)

As a young girl, Mary was very compassionate towards animals. Perl reported that she frequently rescued insects that had been hurt by frost or rain, and nursed them back to health. As an adult, she worked as a librarian in a women's college, and showed the same compassion in becoming a friend and mentor to the students. She invited students to discussion sessions about mathematics and science, and one of these students later wrote; "I found you have given us a power. We can think for ourselves, and find out what we want to know" (p.50). Even as an old lady, during World War I, Mary opened her house to people who needed to "find a quiet place for an hour, away from the turmoil of a country at war and the terrible news in the newspapers" (p.55).

Perl (1993)

Some teachers' comments:

Listening to what children say during discussion offered me a continuous and detailed means of assessing their understanding and progress. Before this session I doubted whether talk/discussion could be obtained in working with a class of thirty-six children. The class was formed into groups, which would discuss mainly on their own. I interacted with these groups by circulating. I controlled a second level of interaction between groups, by calling on spokespersons to report, and drawing in other children appropriately. I reinforce my belief that children need more opportunity to talk about their mathematics.
I learnt that children working together not only have the opportunity to listen and learn from each other, but also to try out some ideas in a non-threatening environment. Every member of a group has the chance of seeing the activity in more than one way than if they were working alone.

Team work can lead to better development of mathematical understanding because of the communication that must occur for the group to function. These activities necessitate that children use all four components of language skills: speaking, listening, reading and writing. Interactions are indeed the heartbeat of the mathematics classroom. Mathematics is learned best when students are actively participating in that learning. One method of active participation is to interact with the teacher and peers about mathematics. (Primary School Teacher)

I chose to work with a group of children about whom I felt I knew very little. I realised that these children could have ability which was not being shown, so I decided to make a more concentrated effort to provide a variety of experiences and activities, to allow some 'non-performing' children to demonstrate their skills. I also recognised the need to discourage a group of 'noisy' boys from putting down the girls and their contributions. A colleague undertook a similar exercise with an older class. She was surprised that she knew the boys better as being more confident and responsive. She intends to investigate this further by asking a colleague to observe her teach to find out whether her suspicions are true that she is responding more to the boys than to the girls.
(Secondary School Teacher)

I was concerned about two things. One was the way I could use praise to develop self esteem. The other thing was the way in which I was involved in my pupils' activities. I chose these issues because I had got into the habit of teaching from the front of the room and responding to the students' answers with comments such as "Okay", "Good", "Sensible". I was also concerned that the girls were outnumbered by boys in the class and there was an underlying assumption that the boys were better than the girls, made particularly evident by a vocal group of boys. I consciously placed myself with different pupils in the classroom and moved to groups when asking or answering questions. I deliberately targeted the quieter children to encourage them to participate in group/class discussions. I developed a repertoire of responses to students' answers, including, "Good thinking strategy," or "Can you clarify that response?" I allowed more response time, focused on permitting girls to respond following incorrect answers and followed their answers immediately by further questions. Although I only had two weeks in which to implement these initiatives, I felt sufficiently positive about the change in quality of the students' responses to warrant continuing this approach.
(Primary School Teacher)

The teachers who wrote the comments above were asked to recommend ideas which they could try in their classrooms to encourage more understanding of those students who may not feel safe to participate as fully as they should or could be.
Recommendations included:

- give continuous encouragement, mainly verbally. Value everybody's responses and have firm rules about interruptions and 'put downs',
- encourage a balance between co-operative and competitive teaching and learning styles,
- demonstrate an 'expectation' for students to participate,
- encourage group work and peer tutoring, particularly on activity-based and problem-solving tasks,
- allow students sufficient time to complete their work,
- encourage different strategies for approaching and solving problems,
- talk to the non-participators about their reasons for lack of participation - perhaps our perceptions are invalid.

References

Bell, E.T.(1937). *Men of Mathematics* (Touchstone Edition, 1986). New York: Simon & Schuster. ISBN 0-671-62818-6 PBK

Lovitt, C. & Clarke, D. (1992). *The Mathematics Curriculum and Teaching Program, Professional Development Package, Activity Bank Volume 2.* Victoria, Australia: Curriculum Corporation. ISBN 0 642 53279 6.

McDonough, A. (1984). 'Fun maths in boxes', in A.Maurer (Ed.). *Conflict in Mathematics Education,* (pp.60-70). Melbourne, Australia: Mathematical Association of Victoria.

Perl, T. (1993) *Women and Numbers*. San Carlos, California: Wide World Publishing/Tetra. ISBN: 0-933174-87-X

Voolich, E. (1993). 'Using biographies to 'humanize' the mathematics class'. *Arithmetic Teacher*, 41(1),16-19.

Appendix:

How Claire's teacher guided her to persevere effectively:

In the circles, arrange the digits 1 to 7 so that the sum of the digits is the same number is any three boxes in a row.

```
     O
  O     O
     O
  O     O
     O
```

My goal is to put numbers in the boxes so that they add up to the same total in each direction.

These are the clues I have:
I can use each number from 1 to 7 once.
I have to find a total for each row
Row 1, row 2 and row 3 must add up to the same total.

One way I can do this is to guess. [Put numbers in randomly.]

That didn't work, so I'll try putting in some different numbers. [Repeat]

I'll try again. [Repeat]

I've tried that way three times and I don't think it is working. I'll try something different. I'll pick a total - 13 will do - and try to make the numbers add up.
2+4+7=13. I'll put 4 in the middle.
3+4+6=13
That leaves 1+4+5. That's not 13.

I'll try again. Pick 11.
1+3+7=11. Put 3 in the middle.
6+3+2=11
That leaves 4=3+5. That's not 11.

I've tried that idea twice and it doesn't seem to be working. I think I'd better try something different. I'll put 4 in the middle because that's the middle number.
Then I'll match the highest and lowest numbers.
The lowest is 1 and the highest is 7, so 1+4+7=12.
That leaves 2 as the lowest and 6 as the highest, so 2+4+6=12.
That leaves 3 and 5, so 3+4+5=12.
That's it.
I found the numbers, but I had to try it three different ways.

We can see that this student was demonstrating the values of *patience, fortitude,* and was remaining *unruffled by ups and downs, successes and failures.* The student tried an idea for long enough to give it a chance to work, but knew when it was a good time to try a different approach.

Now, let us see how Clare's teacher helped her to persevere successfully with the problem on which she had been overpersevering:

> In this second attempt, Clare began by using the strategy of changing numbers at random. After four attempts I suggested that she should try a different strategy. She did not have any ideas, so I suggested that she might look at the problem column by column. She started with the units column and used the strategy "I have to get rid of 4, so change 3 and 1." She continued this strategy into the tens column, "21 - get rid of 6 so change 5 and 1" and the hundreds column, "27 - need 11 - get rid of the bigger numbers, 9 and 7".

Question 2 (with prompting):

> In this magic square, each row, each column and the two long diagonals must each add to the same total and each of the numbers from 1 to 25 is used once and once only. Find the missing numbers.

		25	18	11
3	21		12	
	20	13		4
16	14			23
15		1		17

At first Clare had some problems with this question. She did not read the question before starting. She started on the right track and knew that all rows and columns had to add up to 65. She did not read that each number could only be used once. After she changed numbers three times I suggested a fresh start. Clare selected a different row, but again it was one with two gaps and she encountered the same problems. After another three attempts I intervened. We discussed the problem and Clare worked her way across the square pointing out that they all had two or more missing numbers. It was at this point that she found that one column only had one gap. Clare went on to solve the question with no further difficulty.

Question 3 (with prompting):

> □ x △ = 91
>
> □ - △ = 6
>
> What can you put in □ and △ to make both of these true?

Clare was keen to begin this problem after the confident note on which we had finished the previous question. She began by randomly selecting 12x9, an approximate guess. She then tried verbalising multiplication facts:

10x9=90 - not high enough
11x9=99 - too high.

She then looked unsure of what strategy she should adopt. I provided a hint, that "having one on the end is hard isn't it, not many multiplication facts have a one on the end". This did not prove to be helpful so I suggested a change of approach, looking at the subtraction part of the question. This seemed to help Clare as she began to list subtraction facts beginning with 20 and giving an answer of 6:

20-14
19-13
...
13-7

She then said, "Now I'll go back and see if any of these multiplied equals 91". Thus she was able to select her own strategy and was successful.

Question 4 (without prompting):

> What is my mystery number? If I divide it by 3 the remainder is 1. If I divide it by 4 the remainder is 2. If I divide it by 5 the remainder is 3. If I divide it by 6 the remainder is 4.

In solving this problem, Clare randomly chose a number and divided it by 3, 4, 5 and 6. When this did not produce the desired answer, she tried it two more times, using different random numbers. As these attempts were both unsuccessful, she decided to change her strategy. Clare thought of using a strategy which had been suggested to her in a previous problem solving activity, that of developing a system in her choice of numbers. As the mystery number had to have a remainder of 3 if divided by 5, it had to either end in 8 or 3, as each multiple of 5 ends in either 5 or 0. Clare then wrote down all the numbers that, when divided by 6, had a remainder of 4, as this was the largest of the numbers in the clues and therefore would require going through fewer figures. Of these she looked for the numbers ending in 8 and 3 and divided them by the various numbers in the clues. On her third attempt, she found that 58 was the mystery number.

As can be seen from the above examples, Clare showed increasing *courage, self-confidence* and *independence* in her ability and willingness to *face the challenges* of the problems. At first she did not use the strategy instinctively. On problem 2, it was necessary for the teacher to intervene several times, to prompt her to change strategies and to suggest some alternative strategies. The need for teacher intervention had decreased by problem 3, and by the time she reached problem 4 she was able to recognise for herself when it was appropriate to change strategies.

The following procedure can be useful for teachers to follow, in teaching students how to persevere effectively with a task.

Recommended procedure for
introducing problem solving management model

---

1. Give the student a preliminary problem to solve without guidance. Observe whether the student instinctively used the model.
2. If the model was not instinctively used, introduce it and work through a second problem, demonstrating how to use it.
3. Ask the student to repeat the first problem while you guide him/her to use the model, i.e. prompt the student to change to a different approach after a maximum of three repetitions of the previous strategy.
4. Give two more problems, monitoring the strategy pattern and reminding students, when necessary, to follow the model.
5. Give a fifth problem and ask the student to try to follow the model, changing approach when appropriate, without any prompting from you.
